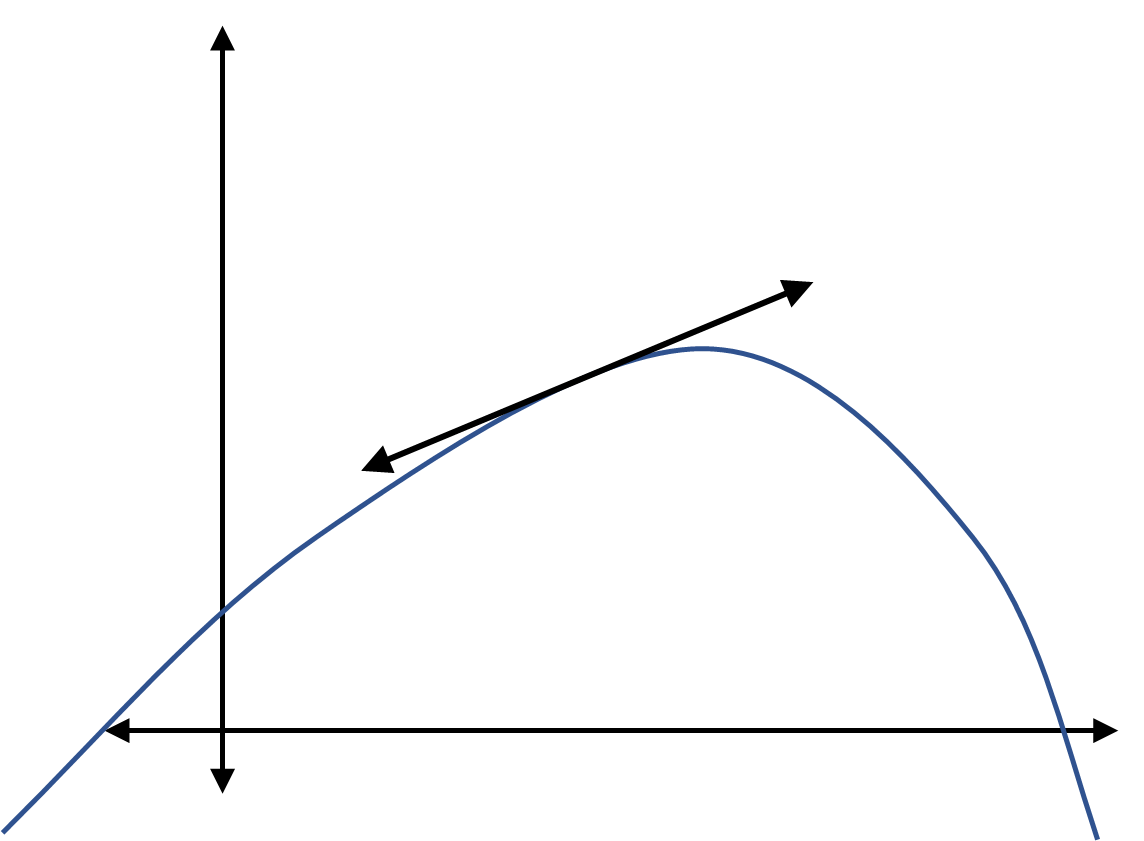
# Tangents – Lines Making Curves

****Tangent – a line that touches the surface of a curve once in the direction of a curve.

Graph 1

Graph 1 shows a tangent to a curve that:

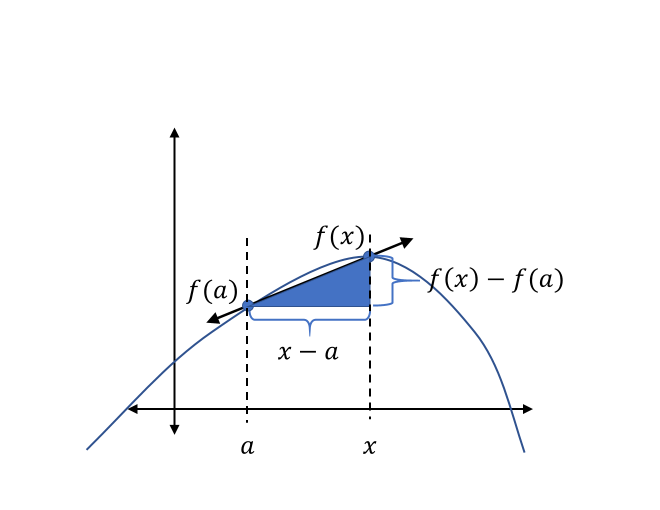
* Touches the curve
* Touches the curve only once
* Goes in the direction of the curve

If you zoom in far enough on any curve, it looks like a straight line.

Take two points on a curve and move them closer and closer together. The line between the two of them approaches closer and closer to the tangent line. (*See pictures below*)

|  |  |
| --- | --- |
|  |  |
|  |  |

Since the distance between the points approaches zero for accuracy, **a limit can find the tangent line**.

A **triangle** shows the tangent line. (Graph 2)

Graph 2

Since the distance between and should approach zero, the **tangent line’s slope is the triangle’s slope**.

**Remember**, is the variable for slope.

Restating the triangle definition above, if is the width of the triangle,

# Derivatives – Rates of Change

Derivative – the measure of how a function’s output changes as its input changes.

The **tangent line is not the same** across a curve (except for a line). This means that one variable () cannot be *the* way to know the tangent of the curve at every point on the curve. The **derivative is like the tangent** (or slope) of the function at every point.

The derivative of any function is another function, .

f’(a) is pronounced “eff-tick of ey.”

Derivatives measure **rates of change** – how much does the function’s output change compared to how much the function’s input changes, at a given point.

“Rate of …” 🡺 “fraction”

“Change” 🡺 “”

“output change compared to…input change” 🡺

Since the input is and the output is ,.

“at a given point” 🡺 “there’s some variable ”

How can the fraction be written? Suppose the function and sample point is . Also, like the definition for tangents above, set a variable to approach zero, to create an imaginary triangle.

🡺

🡺

🡺

Simplifying everything above, .

# Velocity – Derivatives of Position

Velocity – speed with direction.

When objects move, usually their velocity changes. The **velocity does not remain constant** when an object speeds up or slows down or changes direction.

Instantaneous Velocity – the velocity at one precise moment in time.

Because velocity measures how the position changes as time changes, velocity is the derivative of position over time.